N71-36051

NASA TECHNICAL TRANSLATION

NASA TT F-13,569

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Translation of "Avtomodel'naya zadacha o sil'notochnom razryade v plazme," Zhurnal Vychisletel'noy Matematiki i Matematicheskoy Fiziki, Vol. 10, No. 6, 1970, pp. 1447-1457.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 SEPTEMBER 1971

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ABSTRACT. Self-modeling solutions in which the mass of the plasma in the discharge does not vary with time are examined in this paper. It is demonstrated that such solutions exist only in the presence of sufficiently large values of the coefficient of thermal conductivity. The existence of a high-temperature T-layer under certain conditions in the self-modeling regime is established. Certain conclusions are drawn concerning the effect of thermal conductivity on its structure. The analysis of self-modeling solutions is supplemented with numerical calculations of the system of magnetohydrodynamic equations both in the self-modeling and in the "near-self-modeling" region of parametric variations.

1. The investigation of the processes which occur in a plasma during a high-current radiating discharge is connected with the solution of a system of equations of magneto-radiative hydrodynamics (MRHD). In the general case such a solution can be obtained only on the basis of the application of numerical methods. Examples of similar solutions are presented, for example, in the references [1-3]. /1447*

The use of self-modeling solutions in a given problem although associated with definite limitations imposed by the conditions of self-modeling, nevertheless, permits investigation of the separate qualitative aspects of the process and clarification of the nature of its dependence on the parameters such as the coefficients of electrical and thermal conductivity, the current in the discharge, and so forth.

In this paper, self-modeling solutions are investigated in which the mass of the plasma in the discharge does not vary with time. It is established that self-modeling solutions of such a type exist only in the presence of sufficiently large values of the coefficient of thermal conductivity. The

* Indicates pagination in original text.

values of the lower boundary of the region of variation of the thermal conductivity coefficient, where a self-modeling solution exists, are calculated for particular cases.

It is established that under certain conditions there exists a T-layer in the self-modeling solutions [4]. Conclusions are drawn with respect to the effect of the process of thermal conductivity on its structure.

The analysis of self-modeling solutions is supplemented by numerical calculations on an electronic computer of the complete system of equations of MRHD both in the self-modeling and in the "near-self-modeling" region of variation of the parameters.

The self-modeling conditions under which the total plasma energy remains unchanged with time together with a constant plasma mass are analyzed in great detail. In the problem under discussion this constancy of the energy is guaranteed not by the conservative nature of the system but by the equality of the energy fluxes entering and leaving the system.

We note that the self-modeling solutions constructed in this paper are a good test for checking and controlling the accuracy of the numerical methods of solving the system of equations of MRHD. They were used in particular in the working out and preparation of the numerical methods in the reference [3].

2. The dispersion in a vacuum of a plasma formed as the result of the electrical explosion of a wire and its interaction with the magnetic field of the internal currents (see Fig. 1) is discussed. The processes of the heat transfer are taken into account in an approximation of the non-linear thermal conductivity. /1148

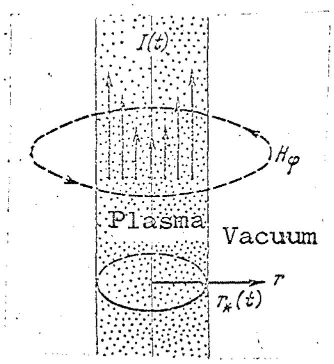

Figure 1.

It is assumed that the length of the plasma filament exceeds by far its diameter and also that axial symmetry holds; the problem is considered in a one-dimensional non-stationary approximation for an infinite cylinder.

The corresponding system of equations of magnetohydrodynamics in Lagrangian mass coordinates in the absolute Gaussian system of units has the form [5]

$$
\begin{gathered}
\frac{\partial v}{\partial t} = -r\frac{\partial p}{\partial x} - \frac{1}{c}\frac{j_z H_\varphi}{\rho}, \qquad \frac{\partial r}{\partial t} = v, \qquad \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial (rv)}{\partial x}, \\
\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{H_\varphi}{\rho r}\right) = \frac{\partial E_z}{\partial x}, \qquad E_z = \frac{c\rho}{4\pi\sigma}\frac{\partial (rH_\varphi)}{\partial x}, \qquad j_z = \sigma E_z, \\
\frac{\partial \varepsilon}{\partial t} = -p\,\frac{\partial (rv)}{\partial x} - \frac{\partial (rW)}{\partial x} + Q, \\
W = -\varkappa \rho r \frac{\partial T}{\partial x}, \qquad Q = \frac{j_z E_z}{\rho}, \\
p = \rho R T, \qquad \varepsilon = \frac{R}{\gamma - 1}T.
\end{gathered} \tag{2.1}
$$

The symbols are as follows: t is the time, r is the Euler variable, ρ is the density of the medium, $x(dx = \rho r dr)$ is the Lagrangian mass variable, v is the longitudinal velocity compenent, p is the pressure, ε is the internal energy, T is the plasma temperature, H_φ is the azimuthal component of the voltage potential of the electric field, E_z is the axial component of the electric field intensity, j_z is the density of the electric currents, σ and $\varkappa$ are the coefficients, respectively, of the electrical and thermal conductivity, Q is the Joule heat generated per unit mass, W is the thermal flux through an azimuthal angle equal to one radian, R is the gas constant, γ is the adiabatic exponent, and c is the velocity of light in empty space; a derivative with respect to the time is Lagrangian.

The equation of state is chosen in the simplest form.

The solution of the problem is sought for a cylinder of unit height in the region $t \geq 0$, $0 \leq x \leq M$, where $M = \int_0^{r_*} \rho r\, dr$ = const. is the mass of the plasma in the discharge taken over a unit height of the plasma filament and included within one radian of the azimuthal angle and $r_*(t)$ is the radius of the boundary between the plasma and the vacuum. The boundary conditions for the system (2.1) are formulated in the following manner: at the center where x = 0 the conditions of symmetry are

$$v(0, t) = 0, \quad H_\varphi(0, t) = 0, \quad W(0, t) = 0, \tag{2.2}$$

and to the right of the plasma-vacuum boundary where x = M ($r = r_*(t)$) /1449

$$p(M, t) = 0, \quad H_\varphi(M, t) = 2I(t) / cr_*(t), \quad T(M, t) = 0; \tag{2.3}$$

I(t) specifies the variation in time of the total current in the discharge.

In the general case of MRHD in the presence in the medium of a nonlinear thermal conductivity the temperature at the boundary of the material with the vacuum is different from zero. The condition T(M, t) = 0 is the limiting case which guarantees the absence of a thermal flux into the system from the vacuum.

Other types of the correct boundary condition for thermal functions are possible, for example, W(M, t) = 0, which corresponds to the case of an electron thermal conductivity or $W(M, t) = \sigma_s T^4$: the plasma filament radiates like a blackbody (σ_s is the Stefan Boltzmann constant). The latter condition leads to supplementary limitations on the conditions of self-modeling obtained below.

To construct a self-modeling solution we will consider the asymptotic phase of plasma dispersion when the effect of the initial data has already disappeared. It is possible to disregard the initial diameter of the plasma filament in comparison with its dimensions in the asymptotic phase and accordingly to assume the initial plasma density to be infinitely large. This permits reducing the number of parameters of the problem which have to be determined.

The coefficients of electrical and thermal conductivity are assumed to be power functions of temperature and density; to achieve greater generality in the derivation of the conditions of self-modeling an explicit dependence of these coefficients on the time is also introduced:

$$\sigma = \sigma_0 T^{k_0} \rho^{-q_0} t^{n_0}, \quad \varkappa = \varkappa_0 T^{k_1} \rho^{-q_1} t^{n_1}. \tag{2.4}$$

The law for I(t) is also given in the form of a power function, namely

$$I(t) = I_0 t^m. \tag{2.5}$$

Furthermore, the case of a constant current m = 0 will also be considered in detail.

We will seek a self-modeling solution of the system of equations (2.1) in which all the functions are given in the form $F(x, t) = F_0 f(s)t^{nf}$, where F_0 is a dimensional constant, $s = x/M$ is a self-modeling variable proportional to the mass variable, and $f(s)$ is a dimensionless function of the self-modeling variable. The self-modeling solutions of this type were investigated in the references [6, 7].

The analysis shows that the conditions of self-modeling in this case reduce to the fulfillment of specific relationships between the parameters of the problem -- the exponents in the power dependences (2.4) and (2.5):

$$m+1=\frac{2k_0+1-n_0}{2(k_0+1)+2q_0}=\frac{2k_1-1-n_1}{2(k_1+q_1)}. \tag{2.6}$$

It also follows from the conditions (2.6) that in the case of the specified relations (2.4) which satisfy the equality (2.6) one can guarantee self-modeling /1450 of the solution because of the corresponding selection of the current law (the quantity m in (2.5)).

For example, in the event of an increasing flux $m > 0$ and the absence of a time dependence in (2.4) and (2.5) ($n_0 = n_1 = 0$) the conditions of self-modeling (2.6) reduce to the inequalities

$$q_0 \leqslant -0.5, \quad q_1 \leqslant -0.5.$$

Thus, the coefficients of electrical conductivity and thermal conductivity should increase with an increase in the density, while in practice the reverse dependence usually occurs. It is true that this dependence is rather weak; it exhibits the most significant effect near the plasma-vacuum boundary. Such a dependence on the density can model the fact that near the boundary with the vacuum the electrical and thermal conductivity decreases more sharply than, respectively, T^{k_0} and T^{k_1}.

If one chooses the constants of the problem M, R, and I_0 as the determining parameters with individual dimensionality, then upon fulfillment of the conditions (2.6) all the unknown functions can be put in the form

$$v(x, t) = \frac{I_0}{\sqrt{M}} \alpha(s) t^m, \quad r(x, t) = \frac{I_0}{\sqrt{M}} \lambda(s) t^{m+1},$$

$$\rho(x, t) = \frac{M^2}{I_0^2} \delta(s) t^{-2(m+1)}, \quad T(x, t) = \frac{I_0^2}{MR} f(s) t^{2m},$$

$$p(x, t) = M\beta(s) t^{-2}, \quad H_\varphi(x, t) = \sqrt{M} h(s) t^{-1},$$

$$E_z(x, t) = \frac{1}{c} I_0 \varphi(s) t^{m-1}, \quad W(x, t) = I_0 \sqrt{M} \omega(s) t^{m-2}, \tag{2.7}$$

$$\sigma(x, t) = c^2 \frac{M}{I_0^2} \tilde{\sigma}(s) t^{-(1+2m)}, \quad \varkappa(x, t) = RM\tilde{\varkappa}(s) t^{-1},$$

$$j_z(x, t) = c \frac{M}{I_0} \zeta(s) t^{-(m+2)}.$$

The equations presented clarify the nature of the dependence of the various functions in the self-modeling stage on the parameters of the problem and the time. For example, the size of the electrical resistance of the plasma per unit length of the plasma filament R_{pl} is calculated in the following manner:

$$R_{pl} = \left[2\pi \int_0^{r_*} \sigma r \, dr\right]^{-1} = \frac{R_0}{c^2} t^{-1}$$

(R_0 is some dimensionless quantity). It follows from this that in the self-modeling stage the resistance of a dispersing plasma decreases with the time, but it does not depend on the mass of the plasma M, or on the nature of the material R, or on the law specifying the variation of the current I_0, m.

The total amount of energy contained in the volume occupied by the plasma /1451 is expressed in the following manner:

$$e(t) = 2\pi \int_0^M \left(\varepsilon + 0.5v^2 + \frac{H_\varphi^2}{8\pi\rho}\right) dx = e_0 I_0^2 t^{2m} \tag{2.8}$$

(e_0 is a dimensionless constant).

If the self-modeling conditions (2.6) are fulfilled, the system of equations of MRHD (2.1) reduces to a system of ordinary differential equations for the dimensionless functions α, β, δ, f, h, λ, φ, and ω:

$$\alpha = (m+1)\lambda, \quad \lambda\beta' = -\frac{\zeta h}{\delta\lambda}, \quad \zeta = \tilde{\sigma}\varphi,$$
$$(\lambda h)' = 4\pi\frac{\zeta}{\delta}, \; (\lambda\omega)' = -\frac{2m}{\gamma-1} f - 2(m+1)\frac{\beta}{\delta} + \frac{\zeta\varphi}{\delta}, \tag{2.9}$$
$$\omega = -\tilde{\varkappa}\lambda\delta f', \quad \beta = f\delta, \quad \tilde{\sigma} = \tilde{\sigma}_0 f^{k_0}\delta^{-q_0}, \quad \tilde{\varkappa} = \tilde{\varkappa}_0 f^{k_1}\delta^{-q_1}.$$

Differentiation with respect to the self-modeling variable is denoted by a prime.

The dimensionless constants $\tilde{\sigma}_0$ and $\tilde{\varkappa}_0$ are expressed in terms of the parameters M, I_0, and R, and respectively σ_0 and $\varkappa_0$, in the following manner:

$$\tilde{\sigma}_0 = \sigma_0 I_0^{2(k_0+q_0+1)}/M^{k_0+2q_0+1}R^{k_0}, \tag{2.10}$$
$$\tilde{\varkappa}_0 = \varkappa_0 I_0^{2(k_1+q_1)}/M^{k_1+2q_1+1}R^{k_1+1}.$$

The boundary conditions (2.2) and (2.3) in the self-modeling form can be written in the following way:

$$\alpha(0) = 0, \quad h(0) = 0, \quad \omega(0) = 0, \tag{2.11}$$
$$\beta(1) = 0, \quad \lambda(1)h(1) = 2, \quad f(1) = 0. \tag{2.12}$$

3. Below we will limit the analysis of self-modeling solutions to the case of constant current (m = 0).

Here with the supplementary assumption $k_0 = q_0 = 0$, a solution is successfully constructed in an analytic form. The coefficients of electrical and thermal conductivity in this case have the form

$$\sigma = \sigma_0 t^{-1}, \quad \varkappa = \varkappa_0 T^{k_1}\rho^{-q_1}t^{-(1+2q_1)}. \tag{3.1}$$

The dependence of σ and $\varkappa$ on the time in (3.1) is very artificial from the physical point of view. However, as the calculations of the system (2.9) - (2.12) show, the main qualitative characteristics of the solution obtained in this most simple particular case are maintained in more general cases for reasonable values of the exponents k_0, q_0, n_0, k_1, q_1, n_1.

It is evident that if for the self-modeling solutions, the current in the discharge is constant, then because of (2.8) the total energy of the plasma and also any quantity which has the dimension of energy, does not depend on the time. The condition of energy constancy is fulfilled in the well known self-modeling solutions of the problem of a strong explosion in the atmosphere [8, 9] where the energy produced at the initial instant does not change in quantity during the rest of the process. In the problem under investigation concerning an electrical discharge in a plasma, the energy constancy is provided not by the conservative nature of the system but by the balance of the electromagnetic energy entering the system and the energy expended as work against the force of the magnetic field and also the energy leaving the system in the form of thermal flux. Evidently in the usual gas dynamics nontrivial self-modeling solutions of such a type are not possible. The presence of supplementary external sources similar to Joule heating is necessary for their existence. /1452

The integration of (2.9) under the condition (3.1), m = 0, and for example, $q_1 > -1$ leads to the following expressions for the dimensionless functions of the velocity α, the pressure β, the magnetic strength h, the temperature f, the density δ, and the thermal flux ω, in terms of the dimensionless radius λ:

$$\alpha = \lambda, \quad \beta = \frac{1}{\pi\lambda_*^4}(\lambda_*^2 - \lambda^2), \quad h = \frac{2}{\lambda_*^2}\lambda, \quad \omega = \frac{\lambda}{2\pi\lambda_*^4}\left(\frac{1}{\pi\tilde{\sigma}_0} - 2\lambda_*^2 + \lambda^2\right),$$
$$f = \{A[(B-1)\lambda_*^2 + \lambda^2](\lambda_*^2 - \lambda^2)^{q_1+1}\}^{1/(k_1+q_1+1)}, \quad \delta = \beta f^{-1}, \tag{3.2}$$

where

$$A = \frac{k_1 + q_1 + 1}{4\tilde{\varkappa}_0 (q_1 + 2)\pi^{q_1+1}\lambda_*^{4(q_1+1)}}, \quad B = \frac{q_1 + 2}{q_1 + 1}\left(\frac{1}{\pi\tilde{\sigma}_0\lambda_*^2} - 1\right).$$

The relationship $S = \int_0^\lambda \beta f^{-1}\lambda d\lambda$ establishes a connection between the dimensionless radius and the self-modeling variable. The quantity λ_* is the dimensionless value of the radius of the plasma vacuum boundary and is determined from the condition that the plasma mass remain constant

$$1 = \int_0^{\lambda_*} \frac{\beta}{f}\lambda\, d\lambda = \frac{1}{\pi\lambda_*^4 A^{1/(k_1+q_1+1)}} \int_0^{\lambda_*} \left\{\frac{(\lambda_*^2 - \lambda^2)^{k_1}}{(B-1)\lambda_*^2 + \lambda^2}\right\}^{1/(k_1+q_1+1)} \lambda\, d\lambda. \tag{3.3}$$

The electric field strength and the current density in the solution are constant:

$$\varphi = \frac{1}{\pi\tilde{\sigma}_0\lambda_*^2}, \quad \zeta = \tilde{\sigma}_0\varphi = \frac{1}{\pi\lambda_*^2}. \tag{3.4}$$

If follows from (3.2) that the pressure is a monotonically decreasing function of the radius, and h increases with an increase in λ. The temperature, f, can be nonmonotonic in λ. The position of its maximum λ_{max} is determined by the expression

$$\lambda_{max}^2 = 2\lambda_*^2 - \frac{1}{\pi\tilde{\sigma}_0}, \qquad (3.5)$$

and the maximum value of f has the form

$$f_{max} = \left[\frac{k_1 + q_1 + 1}{4\tilde{\varkappa}_0 (q_1 + 1)(q_1 + 2)\pi^{q_1+1}\lambda_*^{2q_1}}\left(\frac{1}{\pi\tilde{\sigma}_0\lambda_*^2} - 1\right)^{q_1+2}\right]^{1/(k_1+q_1+1)}.$$

In order that the temperature maximum be located inside the range $0 < \lambda_{max} < \lambda_*$, /1453
the condition

$$\frac{1}{2\pi\tilde{\sigma}_0} < \lambda_*^2 < \frac{1}{\pi\tilde{\sigma}_0}. \qquad (3.6)$$

should be fulfilled.

It follows from the expression for $f(\lambda)$ in (3.2) that the solution has meaning ($f(\lambda) \geq 0$ in the entire region $0 \leq \lambda \leq \lambda_*$) only if the inequality $B \geq 1$ is fulfilled or

$$\lambda_*^2 \leqslant \frac{q_1 + 2}{2q_1 + 3}\frac{1}{\pi\tilde{\sigma}_0}. \qquad (3.7)$$

Thus comparing (3.6) and (3.7), we arrive at the conclusion that the temperature in the solution is not monotonic and its maximum is contained within the range $(0, \lambda_*)$ upon fulfillment of the inequality

$$\frac{1}{2\pi\tilde{\sigma}_0} < \lambda_*^2 \leqslant \frac{q_1 + 2}{2q_1 + 3}\frac{1}{\pi\tilde{\sigma}_0},$$

or equivalently,

$$2 < Re_m^* \leqslant 4\frac{q_1 + 2}{2q_1 + 3}, \qquad (3.8)$$

where $Re_m^* = 4\pi\tilde{\sigma}_0\lambda_*^2$ is the Reynolds magnetic number computed on the basis of the value of the velocity of the plasma-vacuum boundary and its distance from the center. For $Re_m^* \leq 2$ the temperature maximum is always attained on the axis, and $f(\lambda)$ is a monotonically decreasing function. For $Re_m^* > 4(q_1 + 2) / (2q_1 + 3)$, the self-modeling solution has no meaning.

One can conclude from the inequalities presented above that in order for the temperature of the (T-layer) to be nonmonotonic in the solution of the problem of a high-current discharge, it is necessary that the characteristic Reynolds magnetic number be sufficiently large. This statement agrees with the conditions derived in the references [4, 6] for the existence of a T-layer.

We will investigate the nature of the dependence of the self-modeling solution (3.2) on the value of the thermal conductivity coefficient. First, we will discuss the simplest case where $\varkappa$ does not depend on the temperature and the density ($k_1 = q_1 = 0$, $\varkappa = \varkappa_0 t^{-1}$). The quantity λ_* is determined from (3.3) in the explicit form:

$$\lambda_*^2 = \frac{2}{3\pi\tilde{\sigma}_0} \frac{\exp(^1/_4\tilde{\varkappa}_0) - 1}{\exp(^1/_4\tilde{\varkappa}_0) - ^2/_3}.$$

It follows from (3.7) that in this case the self-modeling solution exists only upon fulfillment of the inequality $\tilde{\varkappa}_0 > \tilde{\varkappa}_{01} = 0$.

The condition for nonmonotonicity of the temperature (3.8) can be written in the form

$$\tilde{\varkappa}_{01} < \tilde{\varkappa}_0 < \tilde{\varkappa}_{02}, \quad \tilde{\varkappa}_{01} = 0, \quad \varkappa_{02} = \frac{1}{4\ln 2}. \qquad (3.9)$$

When $\tilde{\varkappa}_0 \geq \tilde{\varkappa}_{02}$, the temperature maximum is located at the center.

We will now discuss the simplest case of a nonlinear thermal conductivity $k_1 = 1$, $q_1 = 0$, $\tilde{\varkappa} = \tilde{\varkappa}_0 T t^{-1}$. The calculations which are similar to those carried out above, lead to the inequality (3.9) with $\tilde{\varkappa}_{01} = 4/\pi$ and $\tilde{\varkappa}_{02} =$ /1454
$4/\pi\ (1 - 2/\pi)^2$; for $\tilde{\varkappa}_0 < \tilde{\varkappa}_{01}$ the self-modeling solution has no meaning, since a region with negative temperature appears in it. The dependence of the self-modeling solution on the coefficient $\tilde{\varkappa}_{01}$ (its values are indicated on the graph for $\tilde{\varkappa}_0 > \tilde{\varkappa}_{01}$ is presented in Figure 2 ($\tilde{\sigma}_0 = 0.02$, $k_1 = 1$, $q_1 = 0$). As $\varkappa_0$ increases, the temperature maximum decreases and shifts nearer to the axis. As $\tilde{\varkappa}_0 \to \infty$, we have $f(0) = 1/3\pi$.

And so it has turned out in two simple particular cases that the region of permissible values for the self-modeling solution is restricted to less than a certain value of $\tilde{\varkappa}_{01}$. An analysis of the solution (3.2) shows that an increase in the exponent k_1, i.e., with an increase in the degree of non-linearity in the thermal conductivity coefficient, the nature of the solution is preserved, and the value of $\varkappa_{01}$ increases.

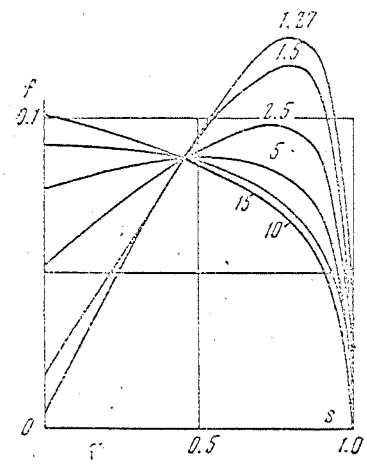

Figure 2.

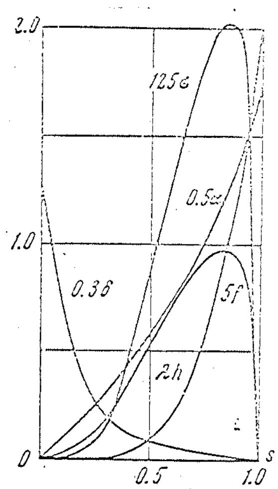

Figure 3.

4. In the case of more general assumptions than the conditions (3.1), the location of the self-modeling solution of the problem of the dispersion of a plasma into a vacuum reduces to the numerical solution of a system of ordinary differential equations (2.9) under the conditions (2.11) and (2.12). It follows from the calculations that the principal qualitative features of the self-modeling solution, which were clarified above analytically, occur in this case. Thus, a typical distribution is presented in Figure 3 for the sought dimensionless functions with respect to the self-modeling variable which were obtained in the calculation of the version of the problem with the following parameter values: $k_0 = {}^3/_2$, $q_0 = 0$, $k_1 = 1$, $q_1 = 0$, $\varkappa_0 = 1.5$, and $\sigma_0 = 0.2$.

Here the plasma conductivity $\tilde{\sigma}$, and also the current density ζ are already not constant, and the maximum of $\tilde{\sigma}$ agrees with the temperature maximum f.

An investigation of the dependence of the solution on the parameter $\varkappa_0$ permits establishing the fact that in this case there exist two characteristic values of the thermal conductivity coefficient $\tilde{\varkappa}_{01}$ and $\tilde{\varkappa}_{02}$. The self-modeling solution occurs only for $\tilde{\varkappa}_0 > \tilde{\varkappa}_{01}$; in the range $\tilde{\varkappa}_{01} < \tilde{\varkappa}_0 < \tilde{\varkappa}_{02}$, the temperature profile is nonmonotonic in s, and for $\tilde{\varkappa}_0 \geq \tilde{\varkappa}_{02}$, the maximum is located at the center.

Thus, even when the self-modeling conditions (2.6) derived on the basis of the usual dimensional analysis are fulfilled, the self-modeling solution does not exist for all values of the thermal conductivity coefficient $\tilde{\varkappa}_0$, although formally, the value of this coefficient does not enter into the self-modeling conditions. /1455

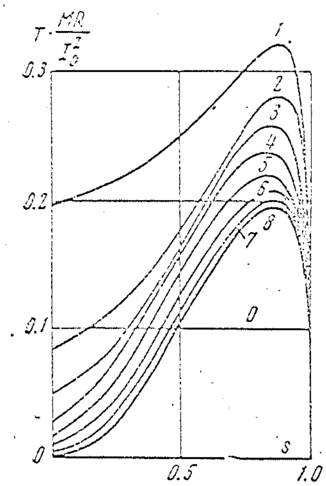

Figure 4.

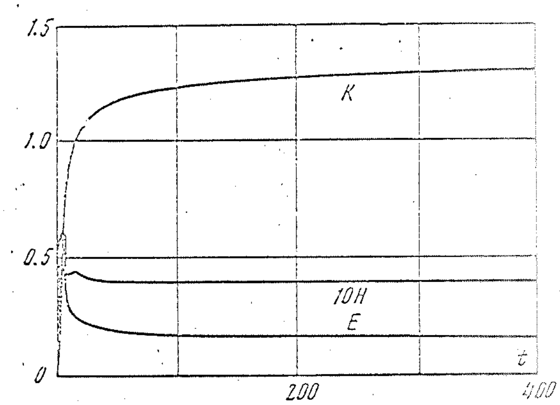

Figure 5.

5. The self-modeling solutions constructed were realized in numerical calculations of the complete system of Eqs. (2.1). The system of differential equations was approximated in the calculations by a uniform completely conservative difference method which was solved by the method of successive elimination [10-12]. The boundary conditions were implemented in agreement with (2.2) and (2.3). The initial conditions were specified in the form of arbitrary functions of the x coordinate which do not coincide with the self-modeling profiles. Such a numerical solution of the problem is presented in Figure 4 for the same parameter values as in Figure 3 with $\tilde{\varkappa}_{01} < \tilde{\varkappa}_0 < \tilde{\varkappa}_{02}$. The temperature profiles are presented at successive instants of time which are selected so that during the time which elapses between them, an identical amount of electromagnetic energy equal to twice the initial enters the plasma. Here, $(MR/I_0^2)T(x, 0) = 0.1$ and $m = 0$.

The solution enters the self-modeling regime as time goes by.

The time variation of the separate types of energy of the plasma are given in Figure 5 in dimensionless form:

internal

$$E = \frac{1}{I_0^2}\int_0^M \varepsilon\, dx,$$

kinetic

$$K = \frac{1}{I_0^2}\int_0^M 0.5\, v^2\, dx$$

and magnetic /1456

$$H = \frac{1}{I_0^2}\int_0^M \frac{H_\varphi^2}{8\pi\rho}\, dx.$$

With an increase in t the values of these quantities and also the value of the total energy tend to their values in the self-modeling solution.

Numerical calculations of the system (2.1) were carried out in the "near self-modeling" region, i.e., with the conditions (2.6) fulfilled, but for small values of the thermal conductivity coefficient, $\tilde{\varkappa}_0 < \tilde{\varkappa}_{01}$. The solutions actually appeared to be nonself-modeling here, and the behavior of the flow parameters did not fit into the framework of the functions (2.7). The non-self-modeling solution has a significantly nonstationary nature; the emergence and growth of a high-temperature T-layer is observed as well as a series of other phenomena usually accompanying it: the formation of a shock wave which propagates toward the axis, a general retardation of the gas, pinching of the plasma filament, and so forth [3, 4].

Summarizing the facts derived on the basis of the analysis of self-modeling solutions and the results of numerical calculations in the"near-self-modeling" region, one can draw conclusions about the effect of the thermal conductivity on the processes which occur during a high-current discharge in a plasma. In the case of a sufficiently small thermal conductivity coefficient ($\tilde{\varkappa}_0 \leq \tilde{\varkappa}_{01}$) a high-temperature T-layer originates and develops. The solution here has a significantly nonself-modeling nature. For example, the temperature of the gas in the T-layer increases, while in the central region it drops.

When $\tilde{\varkappa}_{01} < \tilde{\varkappa}_{0} < \tilde{\varkappa}_{02}$, the effect of the thermal conductivity is already rather strong, due to the fact that the outflow of heat ensures an unusual stabilization of the T-layer: the temperature of the entire mass of the gas varies with time according to one and the same power law.

The high-thermal conductivity coefficient suppresses the T-layer, non-monotonicity of the temperature is not observed in the solution, and its maximum is located on the axis.

6. We will consider one distinctive feature of the analytic solution constructed in Section 3. The temperature maximum which occurs in this solution (both in the self-modeling and in the "near-self-modeling" region) cannot be called a T-layer in the complete sense, for here $\sigma_0 = \sigma_0 t^{-1}$ does not depend on the temperature and the presence of a significant nonlinearity $d \ln \sigma/dT > 0$, is one of the conditions for the formation of this effect. Classical scanning is absent in the present case, since with an increase in t the conductivity of the plasma becomes rather small, and the current density becomes constant along the radius.

Nevertheless, the temperature has a sharply-expressed maximum. The presence of this maximum is explained by the dependence of the amount of Joule heating per unit mass on the density $Q = \zeta^2/\tilde{\sigma}_0\delta$.

The density δ in the present problem drops off because of the intensive dispersion in proportion to the approach to the plasma-vacuum boundary, and the Joule heating correspondingly grows. Such a behavior of the quantity Q
results in the appearance of a temperature maximum whose position, however, /1457
does not agree with the maximum of the Joule heating because of the thermal conductivity processes.

Thus, in the present case, there is a certain limiting degeneracy of the T-layer effect when an inverse relation between the gas dynamic and the electromagnetic processes is absent (the electrical conductivity does not depend on the thermodynamic state of the medium).

If the conductivity is a function of the temperature, then a similar inhomogeneous state can become essential for the development of the T-layer.

So along with the well known skin-effect and the superheating instability, we note one more possible mechanism for the initiation of a T-layer.

The authors are grateful to L. M. Degtyarev and A. P. Favorskiy for useful discussions and also A. A. Ivanov and V. N. Ravinskaya for carrying out the numerical calculations.

Translated from Journal of Computational Mathematics and Mathematical Physics, Vol. 10, No. 6, pp. 1447-1457, November-December 1970. Original article submitted April 8, 1970.

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Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 South Wakefield Street, Arlington, Virginia 22204.